

## Research on Water Utility Revenue Model and Compensation Policy under Uncertain Demand

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### Abstract

With the diversification of both water utility investment and property right structure, it is necessary to establish a scientific compensation mechanism of water conservancy benefit to balance the interests among investors, water users and pertinent sectors which suffer loss. This paper analyzes the compensation policies water management authority imposed on water supply enterprises under uncertain demand, establishes a compensation model with risk preference, explains the implications of risk preference on the decision-making behaviors of water supply enterprises by using numerical analysis method, provides the basis for the water management department to formulate reasonable water resources charge standards and compensation policies. At last, the paper discusses how to implement the water compensation policies according to the characteristics of rural water utilities.

**Key words:** water use uncertainty, risk preference, expected revenue, compensation policy, compensation standard.

### I. Introduction

Water utility investment and operation are faced with some uncertain factors: first, the grain production may be unstable owing to the climatic variables; second, natural disasters affect rural water supply and demand; third, national adjustments of agricultural policies impact on the agricultural water demand; besides, the imposition of water fee and investment returns are influenced by farmers' payment capacity and their insufficient commodity awareness of water. All these risk factors render water utility service exceedingly uncertain and thus affect water utility revenue. In addition, given that water project assumes some public functions such as flood prevention, environmental improvement, irrigation and so on, it's difficult to calculate and quantify its public welfare consumption in the contract. If the interest compensation mechanism is unreasonable, the operating efficiency of these projects invested by the state will be impaired because of managerial dereliction as well as overdue and ineffective compensation. Similarly, in terms of the water programs patronized by the society, the unreasonable investment compensation mechanism is hard to kindle the investors' enthusiasm or to protect the ecological environment in the vicinity of the projects, and thus will undermine the interests of related groups. Therefore, in light of both the uncertainty of water utility revenue and the principle of efficiency and fairness, there is a crying need to establish a compensation mechanism according to the actual cost and proceeds incurred by water engineering maintenance.

Against this background, many scholars have made valuable studies on water compensation problems from different perspectives. Zhang Xiuju and Dong Wenhui, for instance, who analyzed the status of consumptive compensation and its pattern in the public water project, contend that the compensation scope and standard should be defined on the basis of delimiting the boundaries between business assets and the public ones<sup>[1]-[2]</sup>; Shao Wenyan probed into the necessity and measures to carry out water utility compensation<sup>[3]</sup>; Wang Yuanjing explored patterns about benefit compensation and benefit sharing<sup>[4]</sup>; Zhang Shaoqing considered that, in order to ensure the normal operation and maintenance of water utility facilities, various costs of public consumptive compensation in the water utility should be reasonably calculated<sup>[5]</sup>; the Soft Science Research Group of Agriculture Ministry thinks that the implementation of direct subsidies for farmers is conducive to fairness and efficiency<sup>[6]</sup>; Zhu Dongkai, Duan Yuefang studied on the immigrant compensation system of water utility and hydropower engineering<sup>[7]-[8]</sup>; Sun Qingyu deems that the essence of rural water compensation is to ascertain compensation object and compensation standard, namely, a reasonable upper limit of compensation should be based on the structure of revenue sharing, while the lower one on the construction costs and operating costs<sup>[9]</sup>; He Xuefeng analyzed the income of stakeholders of rural farmland water projects<sup>[10]</sup>. Christopher A propounded the franchise contract with flexible term and devised a two-dimensional bidding mechanism<sup>[11]</sup>. Pertinent documentations have already detailed the compensation of consumption induced by the project itself, however, the research on the stakeholders' benefit compensation mechanism, especially on how to establish

compensatory policies in face of uncertain revenue, is by no means deep and exhaustive. Besides, compensation mechanisms for the social funds' non-profit consumption in the water project constructions are not impeccable, which exactly affect the sustainable development of rural water utility and become bottlenecks of rural water project development. Given those foregoing perceptions, this paper introduces the risk preferences of water management authority and water supply enterprise to compensatory strategies, establishes the water management unit's and water supply enterprises' strategy models of compensation with risk preferences, and explicitly expounds the compensatory policies under incertitude demand. According to China's national conditions, it also specifically analyze how to implement the compensatory policies for Chinese water supply enterprises.

## II. Glossaries and Terms

For the sake of convenience, the symbols are defined as follows:  $\phi(x)$  is a probability density function for water demand,  $\Phi(x)$  is a distribution function for water demand,  $r_g$  is a risk preference parameter of water management authority,  $r_w$  is a risk preference parameter of water supply enterprise,  $C_g$  is the cost of water resource,  $C_w$  is the actual fee charged by water management department,  $p$  is the selling price of water,  $Q_0$  is the planned water production to meet consumption,  $b$  is the compensatory price that government provides to water supply enterprise,  $c_l$  is the loss of water enterprise incurred by water shortage,  $x$  is a random demand when the supply water price is  $p$ ,  $B_g$  and  $U_g$  stand for the actual earnings and expected revenue of water management department respectively, while  $B_w$  and  $U_w$  are the actual proceeds and expected revenue of water supply enterprise respectively. Water management department's expected revenue is  $U_g = E(B_g) - r_g Var(B_g)$ , the water supply enterprise's expected revenue is  $U_w = E(B_w) - r_w Var(B_w)$ .

## III. Water Compensatory Policy Model

Suppose that both water resource price and governmental compensatory price are given by the water management department, when the water demand is  $x$ ,

$$\text{For water management department, water cost is } C_g = c_g \cdot Q_0 \quad (1)$$

$$\text{The actual charge collected by water department is } C_w = w_p \square Q_0 \quad (2)$$

The compensation for water supply enterprise provided by water management authorities brings additional cost to water authorities themselves, so the actual earnings of water management department is:

$$B_g = C_w - C_g - R_b = (w_p - c_g - b)Q + b \cdot \min(x, Q_0) \quad (3)$$

$$\text{The actual turnover of water supply enterprise is } R = P * \min(x, Q) \quad (4)$$

The compensation government provides to water enterprise is

$$R_b = b * [Q_0 - \min(x, Q_0)] \quad (5)$$

Since water demand is determined by the market, when the planned water supply of enterprise is less than the users' demand, there will be loss caused by water shortage, and the loss of water enterprise is

$$C_l = c_l [x - \min(x, Q_0)] \quad (6)$$

$$\text{The management cost of water supply enterprise is } C_m = c_m [Q_0 - \min(x, Q_0)] \quad (7)$$

According to above formulas, when the water demand is  $x$ , the actual earnings of water enterprise is

$$B_w = R + R_b - C_w - C_l - C_m = (p + c_m - b + c_l) \min(x, Q_0) - (w_p + c_m - b)Q - c_l \square x \quad (8)$$

Given the above questions, Leader Follower Game can be used to investigate the interactions between water management authorities and water supply enterprise. Water management authority serving as a guidance-providing regulator, specifies the price of water resource and the policy standard of compensation, according to which, while the water supply enterprise, as an object to be regulated, determines its water use plan. The assumption that water management sector will eventually establish the optimal water price and compensatory price on the basis of the reaction function is warranted and universal, because, compared to the water enterprise, water authority has a macroscopical advantage to guide the market. At the same time, because the water market is open, the information about the price in water market, distribution of demand and water cost parameters is symmetrical. As a managerial body of this trade, water authority has all the necessary information to analyze the

reaction function of planned output, which water enterprises expect on the basis of water resource price and compensatory policy, and to make the best decisions. This paper supposes that the water authority and water supply enterprise are absolutely rational, that is they are self-interested and both of them make decisions, before the actual water demand is observed, on the principle of maximizing the expected revenue despite their divergent social welfare objectives.

Suppose that the water demand is in normal distribution,  $\mu$  is the average of distribution function, and standard deviation is  $\sigma$ . Let  $a = \frac{u}{\sigma}$ ,  $b = \frac{Q-\mu}{\sigma}$ ,  $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ ,  $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz$ , through parameter

transformation  $x = \frac{y-\mu}{\sigma}$ , get the averages  $E(x), E(x^2)$  of water demand  $x$  and  $x^2$  respectively, their expressions are

$$E(x) = \mu[\Phi(a) + \Phi(b)] + \sigma[\phi(a) - \phi(b)] \quad (9)$$

$$E(x^2) = (\mu^2 + \sigma^2)[\Phi(a) + \Phi(b)] + (2\mu\sigma - a\sigma^2)\phi(a) - (2\mu\sigma + b\sigma^2)\phi(b) \quad (10)$$

On the basis of them, let  $\Delta = \min(x, Q)$ , we can get the averages of  $\Delta, \Delta^2$  respectively:

$$E(\Delta) = \mu\Phi(a) + (\mu - Q)\Phi(b) + Q/2 + \sigma[\phi(a) - \phi(b)] \quad (11)$$

$$E(\Delta^2) = (\mu^2 + \sigma^2)F(a) + (\mu^2 + \sigma^2 - Q^2)\Phi(b) + Q^2/2 + (2\mu\sigma - a\sigma^2)\phi(a) + (2\mu\sigma + b\sigma^2)\phi(b) \quad (12)$$

The internal relationship between the expected return and the actual return has been defined in the former part, and the expected revenues of water authority and water enterprise have been analyzed in the theoretical model, in accordance with the intrinsic relationships between those definitions, we can convert the actual revenue to the expected revenue with preference. According to the above formulas, the expected revenue of water authority is

$$U_g = E(B_g) - r_g \text{Var}(B_g) = (w_p - c_g - b)Q + bE(\Delta) - r_m \cdot b^2 \text{Var}(\Delta) \quad (13)$$

The expected revenue of water enterprise is

$$U_w = E(B_w) - r_w \text{Var}(B_w) = (p + c_m - b + c_l)E(\Delta) - (w_p + c_m - b)Q - c_l E(\Delta) - r_m(p + c_m - b + c_l)^2 \text{Var}(\Delta) - r_m c_l^2 \text{Var}(\Delta) \quad (14)$$

Comparing the expected revenue functions with preference of water authority and water enterprise, we find that the expected revenue function for water enterprise is more complex. According to the foregoing analysis, firstly, we must determine the reaction function for water enterprise, and then establish a balanced reaction function of water enterprise by following the principle of maximizing the expected revenue in this paper.

On the basis of the above assumptions and definitions, we can obtain the reaction function which includes the variables of the amount of water that the water enterprise plans to supply, water price, government compensatory standard, the cost parameter and demand distribution parameter, namely:

$$[(p + c_m - b + c_l) - 2r_w(p + c_m - b + c_l)^2][Q - E(\Delta)][1 - \Phi(b)] = 2(w_p + c_m - b) \quad (15)$$

This result describes the internal balanced relationship among the water enterprise's water supply plan  $Q$ , water price  $w_p$  and governmental compensatory standard  $b$ . Through the balanced reaction function of water enterprise, water authority gains the balanced water resource price and compensatory standard in accordance with the principle of maximizing the water authority's expected revenue.

#### IV. Analysis of Compensatory Policy

Given the fact that the balanced reaction function of water enterprise is complex and there is no obvious function expression and solution, the balanced solutions are discussed by using the methods of static analysis and numerical analysis in order to perform a specific study of the theoretical model's results. Assume that the water authority's water cost and sewage treatment cost is  $c_g = 2.0$ , compared to the production cost, the loss of water shortage and overproduction of water enterprise can be negligible, thus,  $c_m = c_l = 0$ , water price  $p = 4.0$ , water demand average  $\mu = 200$ , demand standard deviation is  $\sigma = 30,50$ , the risk preference of water authority and water enterprise are respectively  $r_w = 0,1$  and  $r_m = 0,1$ .

According to the hypothesis and the established theoretical model, the wholesale price, compensatory

price, equilibriums of water authority's and water enterprise's expected revenues are obtained as follows:

- (1) When the water authority is risk-neutral, the standard deviation of water demand is 30,  
if the water enterprise is risk-neutral,  $b = 3.96\text{yuan}$ ,  $w_p = 3.98\text{yuan}$ ,  $Q_0 = 200$ ,  $B_g = 1.85$ ,  $B_m = 176.1$ ;  
if the water enterprise is risk-averse,  $b = 3.97\text{yuan}$ ,  $w_p = 3.98\text{yuan}$ ,  $Q_0 = 200$ ,  $B_g = 1.73$ ,  $B_m = 176.2$ .
- (2) When the water authority is risk-neutral, the standard deviation of water demand is 50,  
if the water enterprise is risk-neutral,  $b = 3.94\text{yuan}$ ,  $w_p = 3.98\text{yuan}$ ,  $Q_0 = 200$ ,  $B_g = 1.62$ ,  $B_m = 156.6$ ;  
if the water enterprise is risk-averse,  $b = 3.97\text{yuan}$ ,  $w_p = 3.98\text{yuan}$ ,  $Q_0 = 200$ ,  $B_g = 1.63$ ,  $B_m = 156.7$ .
- (3) When the water authority is risk-averse, the standard deviation of water demand is 30, if the water enterprise  
is risk-neutral,  $b = 0.98\text{yuan}$ ,  $w_p = 3.94\text{yuan}$ ,  $Q_0 = 152.6$ ,  $B_g = 10.3$ ,  $B_m = 136.2$ .  
if the water enterprise is risk-averse,  $b = 3.66\text{yuan}$ ,  $w_p = 1.98\text{yuan}$ ,  $Q_0 = 141.2$ ,  $B_g = 1.32$ ,  $B_m = 131.2$
- (4) When the water authority is risk-averse, the standard deviation of water demand is 50, if the water enterprise  
is risk-neutral,  $b = 0.36\text{yuan}$ ,  $w_p = 3.71\text{yuan}$ ,  $Q_0 = 133.4$ ,  $B_g = 18.1$ ,  $B_m = 112.6$ ;  
if the water enterprise is risk-averse,  $b = 3.46\text{yuan}$ ,  $w_p = 3.92\text{yuan}$ ,  $Q_0 = 95.3$ ,  $B_g = 2.02$ ,  $B_m = 76.3$ .

Form results (1) and (2): when the water authority is risk-neutral and the standard deviation for water demand is low, whether the water enterprise has risk preference or not, the experimental solution of water authority's charging standard is 3.98 yuan, but when the standard deviation of water demand is high, water authority's charging standard will be close to the water supply price. Comparing standard charges of the water authority and those of the water enterprise which have different risk preference, we can find that the charging standard provided by the risk-aversion water authority is lower than that provided by the water authority that is risk-neutral. But comparing the balanced charging standards followed by the water authority and water enterprise which have different risk preferences, we discover that the balanced charging standard provided by the water authority to the water enterprise who is risk-averse is higher than that to the risk-neutral enterprise. It is the degree of risk preference that mainly causes that effect. Because the risk-aversion water authority begrudges gamble, it wins the market with a conservative approach of low charging standard. However, for the water enterprises of risk aversion, water authority attracts them to confront the market bravely by providing an incentive method of high wholesale price.

The balanced data above reflects that the compensatory price  $b$  provided to water enterprise is lower than the charging standard of water authority  $w_p$ , which is reasonable. If the water authority is risk-neutral, no matter what the water enterprise's risk preference is, the compensatory price increases with the increasing standard deviation of water demand. When the standard deviation of water demand increases, the water authority replenish the market risk of water enterprise by raising the compensatory price. When the water authority is risk-averse, the compensatory price decreases with the increasing standard deviation of water demand, which reflects that if the water market risk increases, the ability of water authority and water enterprise to control the market starts to weaken, and the attitude of the risk-aversion water authority to shun risks generates one inevitable result: reducing the compensatory price is the only way to transfer risks. Horizontal comparison of the water enterprises' balanced compensatory prices with different risk preferences shows that the compensatory price provided to risk-averse water enterprise by the water authority is relatively high, the reason for this conclusion is the same as the previous one.

The analysis results show that if the water enterprise is risk-averse, the balanced water plan for enterprise decreases with the increase of demand distribution parameter  $\sigma$ . The water enterprise, who disgests market risk and hopes to reduce the instability caused by it, must calibrate the water price and compensatory standard in order to reduce its water supply plan, which is a normal response of water enterprise. Compared transversely, the balanced supply plan of the risk-averse water enterprise is lower, as a consequence of the negative attitude of water authority and water enterprise to the market risk.

The conclusion is that there is a reversely dynamic relationship among the expected revenue, demand distribution parameter  $\sigma$  and risk preference of water authority, but the development trend of water enterprise's expected revenue is more complicated. When the water authority is risk-neutral, whether the water enterprise is risk-neutral or risk-averse, the water enterprise's expected revenue decreases with the increasing standard deviation of water demand  $\sigma$ ; but if the water authority is risk-averse, whether the water enterprise is risk-neutral or risk-averse, the water enterprise's expected revenue increases with the increasing  $\sigma$ .

## V. Application of Water Compensatory Policies

Due to the fact that water enterprises mostly are owned by the state and the market competition mechanism is not fully established, risk compensatory policies are largely affected by non-economic factors and

personal preference behaviors. Because of these problems, the application of compensatory policies in the water industry has some resistance. In terms of various risks in water supply management, to implement the compensatory policies, we should take different precautionary measures according to the causes and characteristics of risk, such as enhancing the transparency and sharing of information, optimizing contract models, establishing the cost supervision mechanism of water enterprises, adopting the flexible designs and so on.

The managerial objective of water industry is to pursue maximum social welfare, however, due to people's different risk preferences, the utility values of the same risk return and its social welfare levels are different. Therefore, it is the risk preference of the water enterprise regulated by water authority that determines who to assume the risk within the water industry.

1) The information between the water authority and water enterprises is asymmetrical, namely the water authority can only observe the water enterprises' sale performances, but fail to note the degree of water enterprise's effort. According to the relevant incentive theory<sup>[12]</sup>, when the water enterprise is risk-neutral, though the effort is not observable, the optimal effort can still be implemented by providing water enterprise some incentive mechanisms like compensation policy, which means that social Pareto Optimality in the water supply industry can still be achieved.

But if the water enterprise is risk-averse, the inefficiencies of water conservancy investment is revealed by the moral hazard, at this moment, the compensation policy will become dysfunctional. In this case, sharing risks jointly by signing agreements on profits partaking is proved to be the most effective cooperational mechanism.

2) When the water authority has the entire information about the water enterprise's effort level, if the water authority is risk-neutral, and the water enterprise is risk-averse, the water authority can provide a higher water price and compensatory price to water enterprise to reduce the risks of water enterprise, ensure that the water enterprise has a relatively stable income, and make the water authority to enjoy more residual claim and assume most of the risks. If the water enterprise does not reach the required effort level, the water authority can mete out some penalties. As a result, under the condition of Pareto Optimality, stable income can only be achieved by water enterprise's effort level.

If the water authority is risk-averse, and the water enterprise is risk-neutral, then the water authority offers a lower water price and compensatory price to water enterprise, thus the water authority can secure a stable income and make the water enterprise assume most of the residual claim and market risks.

If both water authority and water enterprise are risk-neutral, then put  $r_g = r_m = 0$  into the formula, it's easy to work out the balanced water plan in the case of risk-neutral and to obtain the balanced water price under the compensatory price level, the balanced expected revenues of water authority and water enterprise.

## VI. Conclusion

Since the water management authority and water enterprise belong to different types of organizations, if the effective coordination mechanism is insufficient, there will be some inevitable conflicts of interest between the water authority and water enterprise, which will lead to the decline of social welfare in water industry. According to the results of this paper, the water authority can determine a reasonable water price and compensatory price by taking a comprehensive consideration of water fee and market price, as well as its own risk preference and water enterprise's. Cooperating with the water enterprise, the water authority can realize the overall profit maximization of water supply industry and achieve the ideal win-win situation. If the information is asymmetrical and the water enterprise is risk-averse, the simple compensation policy will lose collaborative effect. Therefore, using the compensation policy model with risk preference, we can have a reasonable analysis about decision-making behaviors of water authority and market behaviors of water enterprise, in order to provide a practical and feasible theoretical tool for decision-makers, especially for those who are in the Chinese water industry, to select and formulate the compensation policies scientifically and achieve the social Pareto Optimality in the water industry.

**Acknowledgement:** National social science fund project support (Serial number: 12XGL010)

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